NASA's astonishing evidence that c is not constant: The pioneer anomaly

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For over 20 years NASA has struggled to find an explanation to the Pioneer anomaly. Now it becomes clear the solution to the riddle is that they have uncovered evidence that c, the speed of light, is not quite a universal constant. Using straightforward Newtonian mechanics NASA's measurements provide compelling evidence that the speed of light depends on the inverse of the square root of the gravitational energy density of space. The magnitude of the Pioneer anomalous acceleration leads to the value of the primordial energy density of space due to faraway stars and galaxies: 1.0838. x 10^{15} Joule/m^3. A value which almost miraculously coincides with the same quantity: 1.09429 x 10^{15} Joule/m^3 derived by J. C. Cure from a completely different phenomenon: the bending of starlight during solar eclipses.

Keywords. Pioneer anomaly, flyby anomaly, speed of light, index of refraction, space energy density, Cure hypothesis.

Introduction

Anderson and collaborators at the Jet Propulsion Laboratory (JPL) have reported [1] an apparent, weak, long range anomalous acceleration of the Pioneer 10 and 11 with supporting data from Galileo, and Ulysses spacecraft. [2, 3] Careful analyses of the Doppler signals from both spacecraft have shown the presence of an unmodelled acceleration towards the sun. By 1998 it was concluded from the analysis, that the unmodelled acceleration towards the Sun was (8.09 +/- 0.20) x 10^{-10} m/s^2 for Pioneer 10 and of (8.56 +/- 0.15) x 10^{-10} m/s^2 for Pioneer 11. In a search for an explanation, the motions of two other spacecraft were analyzed: Galileo in its Earth-Jupiter mission phase and Ulysses in a Jupiter-perihelion cruise out of the plane of the ecliptic. It was concluded that Ulysses was subjected to an unmodelled acceleration towards the Sun of (12 +/- 3) x 10^{-10} m/s^2. To investigate this, an independent analysis was performed of the raw data using the Aerospace Corporation's Compact High Accuracy Satellite Motion Program (CHASMP), which was developed independently of JPL. The CHASMP analysis of Pioneer 10 data also showed an unmodelled acceleration in a direction along the radial toward the Sun. The value (8.65 +/- 0.03) x 10^{-10} m/s^2, agreeing with JPL's result. Aerospace's analysis of Galileo Doppler data resulted in a determination for an unmodelled acceleration in a direction along the radial toward the Sun of, (8 +/- 3) x 10^{-10} m/s^2, a value similar to that from Pioneer 10.

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All attempts at explanation of the unmodelled acceleration as the result of hardware or software problems at the spacecraft or at the tracking stations have failed. A very detailed description of the Pioneer anomaly, of the measurements and of the analysis was given by the JPL team [4]. Conferences have been carried out on the subject, in 2004 [5], in 2005 [6] and in 2008 [7]. Although several explanations have been advanced, no clear consensus exists of the cause of the weak [8] anomalous acceleration experienced by the various spacecraft. With no plausible explanation so far, the possibility has arisen that the origin of the anomalous signal is new physics [9]. Very recently evidence of another puzzling phenomenon, possibly related, has been reported in the motion of other spacecraft, the so-called "flyby anomaly"[10,11]. In this paper the observed anomalous Pioneer acceleration is shown to be an artefact of the Doppler measuring system due to a fundamental change in our concept of the speed of light: a minute change of the index of refraction of vacuum, a function of the gravitational energy density of space as predicted by the Curé hypothesis [12]. It affects c the speed of light in space far from the influence of the sun.

1.- Energy density of space.

By energy density of space it is meant the classical energy density (Energy per unit volume) associated with the potential energy of all forms of force: electric, magnetic, gravitational or any other force in nature. In particular, to be associated to gravitational energy of nearby massive bodies such as the Sun, the Moon and the Earth which can be readily calculated, and to the gravitational energy density produced by the gravitational field of the stars and far away galaxies, not so easily estimated.

The energy density of space associated with the presence of static electric E and magnetic B fields are given by:

$$\rho = \frac{1}{2}\varepsilon_o E^2 + \frac{1}{2\mu_o}B^2 \qquad (1)$$

Where ε_0 and μ_0 are the electric permittivity and the magnetic permeability of space respectively. The equivalent energy density associated with a gravitational field g (m/s^2) is given by

$$\rho = \frac{1}{2}\frac{g^2}{4\pi G} \qquad (2)$$

with G the Universal constant of gravitation. Hence any volume of space is immersed in the universal primordial field of energy ρ^* which includes the immediate gravitational field due to the presence of our own galaxy superimposed on the energy fields of all far-away galaxies. Thus the energy density in the surface of the Earth and in the proximity of the Sun and Moon is given by:

$$\rho = \rho^* + \rho_S + \rho_M + \rho_E \qquad (3)$$

where the energy density due to the Sun ρ_S produced by the gravitational effect of the mass of the Sun M_S is obtained from (2) with $g = GM_S / r^2$

$$\rho_S = \frac{GM_s^2}{8\pi r^4} \qquad (4)$$

Here r is the distance from the centre of the Sun to the point in question. And ρ_M and ρ_E are the energy density due to the gravitational effect produced by the mass of the Moon and Earth respectively and are calculated in analogous manner. The acceleration of gravity g_S due to the Sun at the radius of the Earth's orbit is g_S = 0.00593 m s^{-2}. Hence the Sun's energy density at the Earth orbit is ρ_S = 2.097 x 10^4 Joules/m^3. With the Earth's acceleration of gravity the energy density due to the Earth at the surface is ρ_E = 5.726 10^{+10} Joules/m^3 and the energy density due to the Moon at the surface of the Earth is 0,656 Joules/m^3, a relatively small value, and the universal primordial energy density ρ^* is estimated [12] at 1.09429 x 10^{15} Joules/m^3. This is a value arrived at by an analysis of the deflection of light by the Sun's energy field considered as a refraction phenomenon as reviewed below. [13] J.C. Curé [12, p. 276] explains the energy density of space in the following illuminating words:

> "Every celestial body is surrounded by an invisible envelope of gravitostatic energy caused by the matter of the body and given by Eq. (104). (Our Eq. 4) To proceed with a colorful description, let us assign a yellow color to the sun's gravitostatic energy. Let us picture the background cosmic energy with a bluish color. Now we can see, in our imagination, that the sun is surrounded with a green atmosphere of energy. The green color fades away into a bluish color as we recede from the sun."

2.- Effect of energy density of space

Now let us consider the hypothesis that the speed of light is a function of the energy density of space ρ which in the neighbourhood of the sun and the Earth is determined by a constant background value due to the distant galaxies plus smaller values due to the gravitational presence of the Sun's, Earth's, and Moon's mass as seen by (3) above.
It is assumed the speed is inversely proportional to the square root of the energy density by the use of the Curé hypothesis [12, p 173], which, neglecting the effect of the Moon, is given by:

$$c' = \frac{k}{\sqrt{\rho^* + \rho_S + \rho_E}} \tag{5}$$

This implies that the speed of light *decreases* near the Sun and *increases* far away from the sun. We may then assign an index of refraction n to space such that n = 1 in vacuum space near the Earth, as we usually do, and assign an index $n' < \mathbf{1}$ far away from the Sun, in deep space, where the speed of light c' is greater and is given by:

$$c' = \frac{c}{n'} \tag{6}$$

so that the index of refraction there is $n' = c/c'$.
Using (5) we may write expressions for c and c' and obtain the index of refraction, n', far away from the Sun in terms of ρ_{S1AU} the energy density of the Sun at the distance of the Earth: one Astronomical Unit (r = 1 AU), ρ_E the energy density of the Earth at the surface, ρ_{Sfar}, the energy density of the Sun, relatively far away but in the vicinity of the Sun and ρ^* the interstellar primordial energy density in the vicinity of the Sun [14] as:

$$n' = \frac{\sqrt{\rho^* + \rho_{Sfar} + \rho_{Efar}}}{\sqrt{\rho^* + \rho_{S1AU} + \rho_E}} \tag{7}$$

Strictly speaking, relation (7) should contain in the numerator and denominator the gravitational energy density due to all massive bodies, the other planets, satellites asteroids, etc. However, their contribution is negligible due to the $1/r^4$ factor in the energy density, unless n' is being calculated near a planet.

At this point it is fitting to address the order of magnitude of the quantities being discussed. With n = 1 at the Earth at 1 AU from the Sun, the index of refraction n' further away from the Sun is dependent on the relative magnitudes of the energy density values that enter into Eq. (7), i.e. the relative value of the Sun's energy density, the Earth's energy density and the primordial energy density ρ^* of space due to the stars and far-away galaxies.

If we plot relation (4) we find that ρ_S falls of rapidly as we go away from the Sun, see Fig. 1, and it becomes negligible for distances of say r > 10 AU compared to the universal primordial energy density estimated by Curé [12, p 279] at 1.094291 x 10^{15} Joules m^{-3}. Entering these values into (7) we find that n' is smaller than one for r > 1 AU, and it is also smaller than one for r < 1 AU due to the energy density of the Earth which, near the surface, is much greater that the sun's energy density. But the numerical value of n' is very nearly one, differing only by a very small amount (see Fig. 2 and Table I). Hence the values of the speed of light calculated at different distances from the Sun changes little from the accurately measured value on the surface of the Earth at a distance of 1 AU from the Sun. These minute changes in the speed of light or of the index of refraction of space are consistent with the tiny magnitudes of the accelerations reported by the Pioneer anomaly.

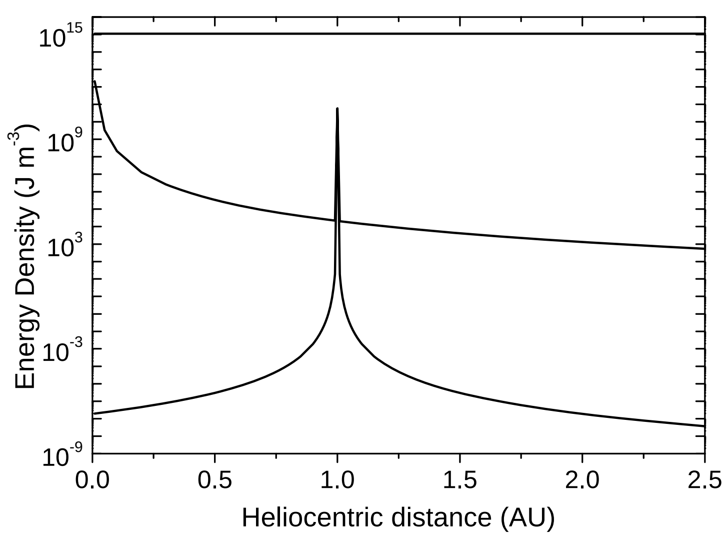

Figure 1. Energy density of space along a radial line Sun–Earth as a function of distance from the sun. Top line, energy density due to the stars. Middle line, Sun's gravity + Earth. Bottom line, energy density due to Earth.

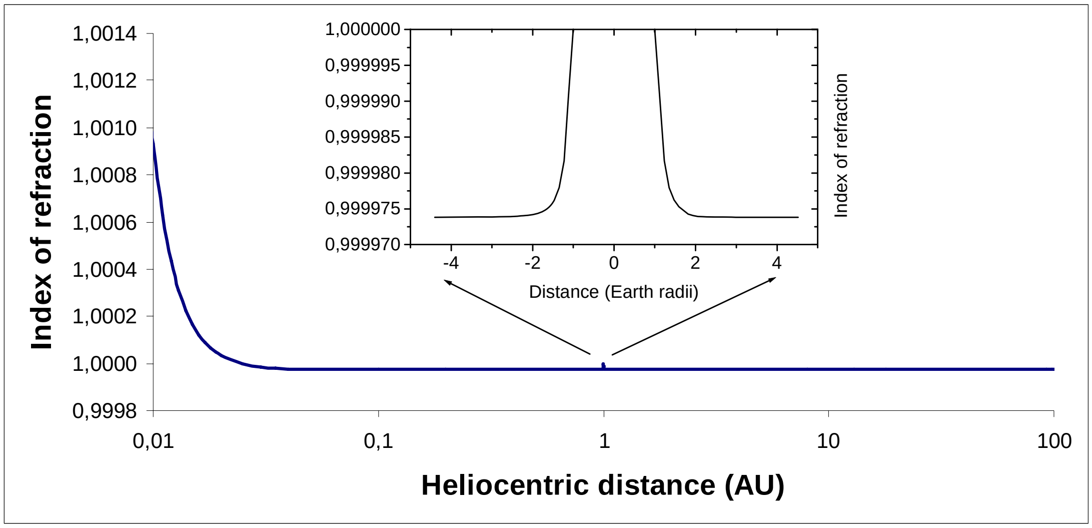

Figure 2. Index of refraction of space as a function of heliocentric distance along a radial line Sun–Earth calculated with Eq. (7) [15]

Table I. Values of the index of refraction n' in the surface of the planets and the Moon. The value of ρ^* = 1.09429 x 10^{15} Joule/m^3 was used in evaluating n' with equation (7).

Planet	Mercury	Venus	Earth	Mars	Jupiter	International space station
n'	0.99997382	0.99999527	1.00000000	0.99997758	1.00014145	0,99997869
Planet	Saturn	Uranus	Neptune	Pluto	Moon	
n'	1.00000349	0.99999524	1.00000737	0.99997385	0.99997454	

In the calculations above we have used for ρ^*, the energy density of the far-away stars, the value calculated and given by Cure [12, p 279]. However, with our knowledge of the energy density of the Sun and Earth, relation (7) for the index of refraction n' may be used to determine the primordial energy density of space, ρ^*, if we do an *independent measurement* of the index of refraction of space, n', far away from the Sun. This is done further down.

Solving (7) for ρ^* we get:

$$\rho^* = \frac{\rho_{Sfar} + \rho_{Efar} - n'^2\left(\rho_{S1AU} + \rho_E\right)}{n'^2 - 1} \tag{8}$$

In this relation n' is the index of refraction at the distance where $(\rho_{Sfar} + \rho_{Efar})$ is calculated.

3.- Doppler Effect.

The frequencies of signals received from spacecraft are affected by their movement through the Doppler Effect. In fact the first order Doppler Effect is normally used to determine the speed of distant spacecraft. An accurate oscillator "clock" on board emits a signal in the form of an electromagnetic wave at a base frequency f_o. If the spacecraft moves at a velocity, v, relative to the receiving station the frequency f of the clock as perceived by the receiver is shifted from f_o by an amount Δf :

$$\Delta f = f_o - f = f_o\left(\frac{v}{c}\right) \qquad (9)$$

Hence

$$f = f_o - f_o\left(\frac{v}{c}\right) = f_o\left(1 - \frac{v}{c}\right) \qquad (10)$$

This is the frequency received when v is in the direction away from the receiver, i. e. the signal of a receding spacecraft is Doppler-shifted towards lower frequencies (red-shifted). The reverse occurs if the spacecraft moves towards the receiver, in which case the received signal is Doppler-shifted towards higher frequencies (blue-shifted).

Above it is assumed a "clock" on board for clarity in the argument. However, in the case of the Pioneer spacecrafts this is not true. The signals transmitted by the Pioneer spacecrafts are re-transmission of Earth-sent signals. Assume the frequency transmitted from Earth is f_e, the spacecraft is in motion relative to Earth hence the frequency of the signal received at the spacecraft for retransmission is not f_e but rather a Doppler shifted frequency f_o. The shift is given by a relation analogous to (9): In the spacecraft frame of reference Earth is receding with speed v. Hence the signal received is Doppler shifted by an amount Δf_s

$$\Delta f_s = f_e - f_o = f_e\left(\frac{v}{c}\right)$$

Solving for f_o we obtain a relation like (10):

$$f_o = f_e\left(1 - \frac{v}{c}\right)$$

Which substituted in (10) gives:

$$f = f_e\left(1 - \frac{v}{c}\right)^2 = f_e\left(1 - \frac{2v}{c} + \frac{v^2}{c^2}\right)$$

Neglecting the second order term the Doppler-shifted frequency f received on Earth due to the spacecraft in motion with speed v is

$$f = f_e\left(1 - \frac{2v}{c}\right)$$

and the change relative to the Earth-sent frequency is:

$$\Delta f = f_e - f = f_e\left(\frac{2v}{c}\right) \qquad (11)$$

4.- Doppler effect with c affected by the energy density of space

Let us now consider a Pioneer spacecraft far in deep space, in a region of space where $n' < \boldsymbol{1}$ re-transmitting an Earth-sent base frequency f_e and moving away from a receiver station at a hypothetical *steady (constant) velocity* v.

The frequency f and the frequency shift Δf of the signal perceived by a receiver will not be given by relation (11) above but rather by:

$$\Delta f' = f_e - f' = f_e\left(\frac{2v}{c'}\right) \tag{12}$$

The primed variables are the values affected by the fact that the speed of light c' in the region of the spacecraft is different because there the index of refraction is n'.

Substituting $c' = c/n'$ we get:

$$\Delta f' = f_e - f' = f_e\left(\frac{2v}{c}\right)n' \tag{13}$$

The meaning of Eq. (13) is that the frequency shift perceived at the receiving station is smaller because $n' < \boldsymbol{1}$. Accordingly it would correspond to a smaller Doppler shift and hence interpreted by an observer, *unaware of the value of* n', as due to a receding velocity of the spacecraft that is *smaller* that it actually is.

5.- Effect of Gravity on speed of spacecraft

A spacecraft that is receding into deep space away from the Sun does not move with a constant velocity. This is because it is affected by the gravitational attraction of the Sun. The effect is that the receding spacecraft is affected by a change of speed towards the Sun which is equal to the magnitude of the Sun's acceleration of gravity at the position of the spacecraft. The acceleration is in the direction of the Sun which is approximately in a direction opposite to its receding speed.

For a deep space probe spacecraft the acceleration a is given from Newton's second law by $a = F/m$ with F the gravitational force of the Sun on the spacecraft and m the spacecraft mass. F is given by Newton's relation: $F = GM_S m/r^2$ with G the universal constant of gravitation, 6.67300×10^{-11} m^3 kg^{-1} s^{-2}, and M_S the Sun's mass, 1.98892×10^{30} Kg, hence the acceleration of the spacecraft is:

$$a = \frac{GM_s}{r_s^2} \tag{14}$$

where r_S is the radial distance from the spacecraft to the centre of the Sun.

The speed of the spacecraft is time dependent and is given by: $v = v_0 - at$ with v_0 the speed at some time $t = 0$, and a the acceleration given by (14):

$$v = v_o - \left(\frac{GM_s}{r_s^2}\right)t \tag{15a}$$

If we wish to take into account the gravitational force of the Earth, we must include a term similar to (14):

$$v = v_o - \left(\frac{GM_s}{r_s^2}\right)t - \left(\frac{GM_e}{r_e^2}\right)t \tag{15b}$$

Where M_e is the mass of the Earth 5.98 x 10^{24} Kg and r_e is the distance to the spacecraft from the centre of the Earth.
Now let us consider the effect on the Doppler signals on a spacecraft whose speed is modified by the gravitational attraction of the Sun and the Earth. The speed in not constant but rather a function of time given by Eq. (15) above, hence it is Doppler shifted by:

$$\Delta f' = 2 f_e \frac{(v_o - \frac{GM_S t}{r_s^2} - \frac{GM_e t}{r_e^2})}{c} n'$$

With $\Delta f'$ being a function of time, the time rate of change of the Doppler shifted signal is:

$$\frac{d\Delta f'}{dt} = -\frac{2 f_e n' G}{c}\left(\frac{M_S}{r_s^2} + \frac{M_e}{r_e^2}\right) \qquad (16a)$$

However, if we neglect the change of the speed of light due to the energy density of space we would have the previous relation with n' = 1 as follows:

$$\frac{d\Delta f}{dt} = -\frac{2 f_e G}{c}\left(\frac{M_S}{r_s^2} + \frac{M_e}{r_e^2}\right) \qquad (16b)$$

Hence the "Excess" Doppler shift E_D (Hz/s) due to the effect of the energy density of space is given by the difference between these two relations:

$$E_D = \frac{d\Delta f'}{dt} - \frac{d\Delta f}{dt}$$

Or

$$E_D = \frac{2 f_e G}{c}\left(\frac{M_S}{r_s^2} + \frac{M_e}{r_e^2}\right)(1 - n') \qquad \text{(Hz/s)} \qquad (17)$$

Relation (17) gives the "Excess" Doppler signal that is detected by a receiving station on Earth and interpreted as an anomalous acceleration towards the Sun due to the effect on the Doppler frequency by the higher speed of light in the interstellar medium as compared with the speed of light, c, on Earth.
Upon examination of Eq. (17) it is seen that the term in the parenthesis, $(1 - n')$, is very small owing to the fact that n' is smaller than one, but very near to one. At a distance of 20 AU from the sun this term is equal to 0.0000572. The term on the right of Eq. (17), excluding $(1 - n')$, is the factor $(2 f_e / c)$ times the gravitational acceleration of the Sun and the Earth at the distance r, i.e. it is the drift of the Doppler signal due to the gravitational acceleration at that point. An acceleration which is mainly due to the Sun.
The Pioneer anomaly reported as a weak acceleration, a, toward the Sun is calculated from the time rate of change of the Doppler shift, Eq. (11):

$$E_D = \frac{d\Delta f}{dt} = \frac{2 f_e}{c}\frac{dv}{dt} = \frac{2 f_e}{c} a \qquad \text{(Hz/s)}$$

Hence the anomalous acceleration is:

$$a = G\left(\frac{M_S}{r_s^2} + \frac{M_e}{r_e^2}\right)(1 - n') \qquad (\text{m/s}^2) \qquad (18)$$

With n' given by Eq. (7).

Examination of (18) and (7) shows that the only unknown parameter is ρ^*, the primordial energy density of space due to the stars and far-away galaxies. Hence it is possible to predict the magnitude of the Pioneer anomaly with ρ^* as a single adjustable parameter.

One may use Eq. (18) in the following way:
With an accurate empirical value of the "Excess" Doppler shift, E_D, measured by Anderson and collaborators we can calculate what is the index of refraction n' for a particular position of the deep space probes. Solving Eq. (17) for n':

$$n' = 1 - \frac{E_D c}{2 f_o G\left(\frac{M_s}{r_s^2} + \frac{M_e}{r_e^2}\right)} \tag{19}$$

This then allows determination of the speed of light c' in that position with $c' = c/n'$. It then allows an independent calculation of the energy density of space ρ^* due to the far-away stars or primordial energy field with relation (8) assuming the Curé hypothesis given by relation (5).
Another way Eq. (18) can be used is to calculate a relation for the "Excess" acceleration of the spacecraft as a function of distance from the Sun and Earth using an assumed value for ρ^* and all the other known astronomical values.

6.- Results
Here are shown the numerical results of calculations using the theory above.
i.- With the use of (19) and of data published [Ref. 4, p 15] of the frequency used in the transmission to the pioneer spacecraft of f_e = 2295 MHz and the "Excess" Doppler shift, E_D, a steady frequency drift of $(5.99 \pm 0.01) \times 10^{-9}$ Hz/s from the Pioneer 10 spacecraft [4, p 20, 18, p 4], we calculate with (19) that the index of refraction n' at 20 AU from the Sun is:

$$n' = 0.9999735679^\dagger \tag{20}$$

With this value the accepted speed of light measured on the Earth at 1 AU as c = 299792458 m/s becomes at about 20 AU the slightly higher value of c' = 299800382 m/s . The value (20) is the result of an empirical measurement of the index of refraction of space at 20 AU by NASA's careful measurements [16] of Pioneer signals.
With this value and the use of Eq. (8) we can calculate the primordial energy density of space ρ^*, using the Sun's and the Earth's energy density at 1AU and at 20 AU. The value calculated is:

$$\rho^* = 1.0838. \text{ x } 10^{15} \text{ Joule/m}^3. \tag{21}$$

This value coincides with the value of ρ^* = 1.09429 x 10^{15} Joule/m^3 calculated by Curé on the basis of an entirely different phenomenon: The bending of starlight rays by the gravitational field of the Sun. We outline here the calculation done by Jorge Céspedes-Curé of this important property of space [12, p. 279]. It consists of using the hypothesis of Eq.

† To 10 digits, although rightmost digits are not significant due to imprecision of E_D

(5) interpreted as a change of the index of refraction of space, and using the analysis carried out by Prof. P. Merat [17] in 1974 [12, p 274] for 297 starlight deflections measured in 9 observations of 6 solar eclipses. With the results of Merat's analysis of the astronomical observations of the bending of starlight rays by the gravitational field of the Sun, Curé determines ρ^* the energy density of space due to the far-away stars and galaxies.

ii.- Here the value calculated by Cure for ρ^* is assumed. Together, with all the other astronomical data, it is used in relation (7) to calculate, at a distance of 20 AU, the index of refraction n'. This allows with the use of (18) to determine the anomalous acceleration. The numerical result is 7,84 x 10^{-10} m s^{-1} This value is plotted in Fig.3 for comparison with values reported by the JPL team [4, Table I, p 24]

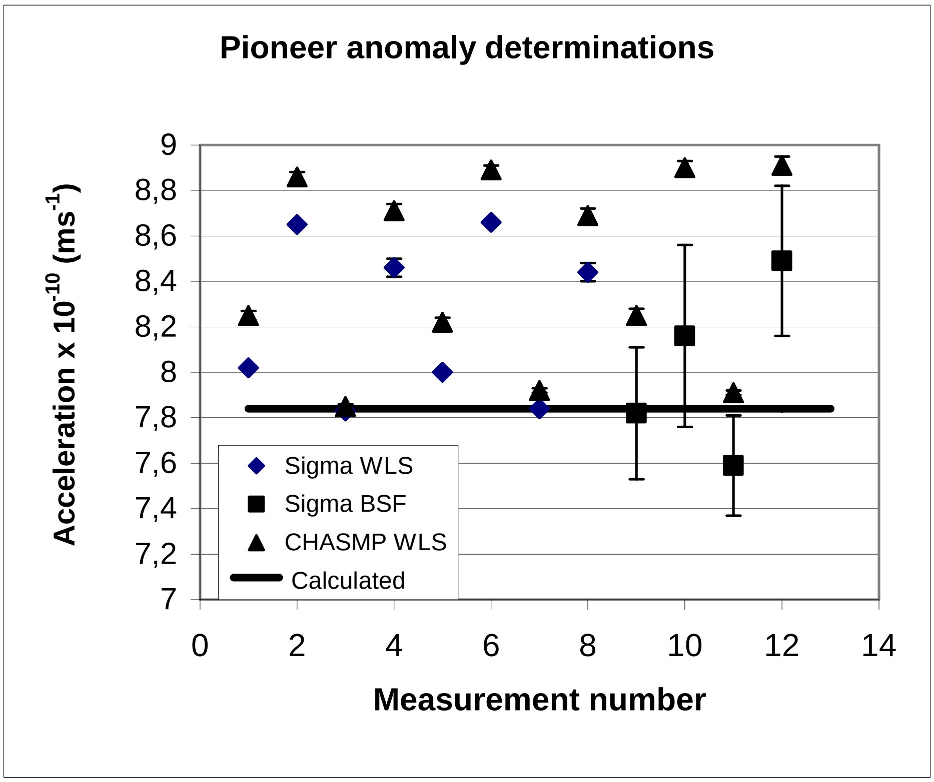

Figure 3. Comparison of calculated and measured values [4, Table I, p 24] of the anomalous acceleration.†

7.- Discussion.

As the dispersion shown in Fig.3 indicates, the measurements of the Pioneer anomaly have a large scatter. They are of the same order of magnitude of the errors in the measurements and with this imprecision they do not show a variation with the distance to the sun. However, considering the wildly different, magnitudes of the data that enter the relations

† Error bars given are formal calculation errors. The much larger deviations of the results from each other indicate the sizes of the systematics that are involved [4, p 24]

(Eq.(8) and (19)) used to calculate the space energy density given by (21) (Gravitational constant, mass of the Sun and Earth, both masses squared, speed of light, distance of Sun and Earth to spacecraft squared, both distances to the forth power, the frequency chosen by NASA for Doppler measurements and the frequency drift of the Pioneer transmissions) it seems miraculous that the calculation of the energy density ρ^* in deep space differs by less than 1 % of the value predicted by Curé on the basis of a completely different phenomenon: starlight deflection by the Sun. This is not a freak coincidence but rather a strong proof of the validity of the Cure hypothesis.

The puzzling fact that the anomalous acceleration shown by Pioneer is not observed in the planets may be explained: The anomalous acceleration is not real, it is an artefact affecting Doppler measurements of bodies which are in a place where the index of refraction $n' \neq 1$ and are in relative velocity or acceleration to Earth-bound observers. A Doppler probe on the surface of the planets will show an anomalous value because the energy density of space there is different from the energy density on the surface of Earth. Hence the index of refraction n' on the surface of planets differs from Earth. Table I shows the results of calculating n' with the use of Eq. (7). The values close to 1.0 being caused by the local gravitational energy density being not so different from the surface of the Earth. Values of n' above one indicate that a Doppler probe would show an anomalous acceleration in the direction opposite to the Sun and would be equal to the factor $(1-n')$ times the real relative acceleration of the planet.
The recently reported Flyby anomaly [9, 10, 11, 18] is an observation which can be explained qualitatively in view of the radial change of the index of refraction of space portrayed in Fig. 2 and its effect on the Doppler signals used to measure it. Quantitative predictions require complex analysis and are being attempted.

8.- Conclusion

A neo-Newtonian explanation of the Pioneer anomaly has been found. This is done with the Curé [12, p. 173] hypothesis that the speed of light at a site depends on the local space energy density predicted by Newton's universal law of gravitation. With this hypothesis it has been possible to deduce in a simple manner the empirically observed phenomenon of the Pioneer anomaly qualitatively and quantitatively. Additionally with the theory developed one is able to calculate the energy density of space produced by the rest of the Universe in the neighbourhood of the Sun. The value obtained (1.0838. x 10^{15} Joule/m^3) coincides very closely with a value (1.09429 x 10^{15} Joule/m^3) deduced by J. C. Curé [12, p. 279] on the basis of the empirical measurement of light bending by the Sun observed during solar eclipses.
The anomalous acceleration does not exist. Pioneer 10 and 11 as well as Galileo and Ulysses spacecraft are moving according to Newton's universal law of gravitation or according to Einstein's General Theory of Relativity which coincide in this respect. The anomaly is found to be due to the effect on the Doppler signals by the index of refraction of space, which is to say the variation of the speed of light due to the energy density of space predicted by the Curé hypothesis.
For further verification of the Curé hypothesis it is suggested:
1.- Careful analysis of measurements done on the Pioneer spacecraft in the early stages of the flight, from launch to about 20 AU. Fortunately there are plans at JPL, motorized by S.

G. Turyshev, to reanalyze all the data taken of the Pioneer missions, which have now been preserved. [9]
2.- A very accurate and precise measurement of the speed of light in the international space station or surface of planets to verify the prediction shown in Table I

NASA's careful measurements and the Curé hypothesis that the speed of light at a site depends on the local space energy density which explain it, have profound implications for physics and cosmology. A lot of other astronomical data needs to be examined in this context. Its acceptance on the basis of the evidence supplied by an explanation of the Pioneer anomaly and the light bending by the Sun obliges a careful revision of the interpretation of data used by Hubble to derive the hypothesis of the expansion of the universe and all the theoretical predictions which follow.

Acknowledgements
I would like to thank my colleagues Haydn Barros, Imre Mikoss and Guillermo Chasín for helpful discussions during the development of this work and Gabriel Bernasconi for recent literature on the Pioneer anomaly. Also thank Simon E. Greaves for independent calculation of the numerical values and to Jorge C. Curé for pointing out that the Pioneer anomaly could be explained on the basis of work in his book.

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