The index of refraction of quasi-empty space

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Abstract

An early contention of A. Einstein regarding the influence of the magnetic field on the propagation of light is examined. It is found that the contention is compatible with the Cespedes-Cure hypothesis and the idea that the index of refraction of empty space containing magnetic energy density (Quasi-empty space) is modified by the total energy density of space mainly gravitational due to the far away stars and galaxies. Compelling evidence for the Cespedes-Cure hypothesis are coinciding values of the gravitational energy density of space ρ^* due to the far-away stars and galaxies, calculated on application of the hypothesis to the Astronomical Lensing phenomenon and to the Pioneer Anomaly problem. Consequences include a revision of Doppler effect-derived radial velocities that lead to overestimation of recession velocities at locations with a decreased gravitational energy density. The radially decreasing mass density observed visually in galaxies may explain the "Flat rotation curve of galaxies" and an assumption of a finite universe with radially decreasing mass density leads to a plausible explanation of the deviation from linearity of Hubble's law: the accelerated expansion of galaxies at extreme distances. Three possible earth bound laboratory experiments to provide additional evidence of the A. Einstein contention and the Cespedes-Cure hypothesis are presented.

Keywords: Speed of light, Index of refraction, Hubble's law, Universal expansion, Dark Matter, Fundamental physics

1. Introduction

The unsuccessful search for direct experimental evidence of the mathematically discovered dark matter and dark energy as well as the recent observation of extreme deviations from linearity of Hubble's empirical law, or acceleration of the universal expansion by very far away stars and galaxies has prompted C. J. A. P. Martins, in a recently invited report [1] to comment that "...new physics is out there waiting to be discovered." Perhaps the seeds of this elusive new physics may be found in reports of the last century such as the following: Nobel Price winner Peter Kapitza in his book [2] relates a request made to him by Albert Einstein in 1930 to measure the effect of high magnetic fields on the propagation of light [3]. Kapitza had developed, in his work at Cambridge, the most intense magnetic fields worldwide. In 1930, the special and general relativity theories were reaching maturity, however Einstein's intuition let him to formulate the contention that magnetic fields could affect the speed of light. Many years before, the contention that the propagation of light is affected by a magnetic field had been advanced by Albert Einstein in 1894 or 1895 as described by Gerald Holton [4].

The magnetic energy density ρ at a point in space where the magnetic field intensity is **B**, is calculated by:

$$\rho = \frac{1}{2\mu_o} B^2 \tag{1.1}$$

Hence the remark by Kapitza [on ref 3]: "since the effect should depend on the square of the magnetic field intensity" gives a clear indication to Einstein's thoughts at the time, namely, that the *magnetic energy density* is the physical property that would affect light propagation. To our knowledge, an experiment testing to sufficient accuracy the effect of a magnetic field on the speed of light has not been done¹. Experiments to determine the lower limit of the photon charge by attempting to detect a light beam deflection produced by a magnetic field [5] or by an electric field [6] have been designed with a beamfield geometry that does not provide a test of Einstein's contention. The assumption that the speed of light is constant in vacuum does not collide with the fact that "pure vacuum" where light speed is assumed constant, is difficult if not impossible to attain. Even in interplanetary areas far away from any massive objects, space is filled

¹ On a suggestion of the late Jorge Cespedes-Cure, the author performed an interferometer experiment in the early 1980s with a 2 Tesla magnetic field with inconclusive results. In the light of this author's current knowledge, this experiment was incapable of detecting the effect by several orders of magnitude.

by intergalactic magnetic fields, is transverse by electromagnetic radiation, by elementary particles and permeated by the gravitational fields from nearby planets or stars and by the far away stars and galaxies. Hence the term quasi-empty space is here defined as an area where usual vacuum conditions prevail but where gravitational, magnetic or electric fields are present determining a nonzero energy density of space. In this paper we elaborate on Einstein's contention, widening it to include the effect on the propagation of light by the energy density of space due to gravitational masses such as the Sun, Earth or far away stars and galaxies. Compelling experimental evidence for the theory and some consequences are presented, ending in the suggestion of three experiments that might provide further support.

2. The Cespedes-Cure hypothesis

The phenomenon of light bending due to the presence of massive objects was predicted by Einstein and the first confirmation was obtained in the 1919 observations of the eclipse of the Sun by Eddington. [7]. Further evidence of light bending by massive bodies has been observed astronomically and the phenomenon is now called "Astronomical lensing" [8]. Jorge Cespedes-Cure in his book, [9] gives an alternative explanation to the lensing phenomenon observed during eclipses [9, Sec. 6.7, p. 273]. He assumes that the gravitational energy density in a given point of space determines the index of refraction at that location.

The Cespedes-Cure hypothesis consists in an assumption that c the speed of light is inversely proportional to the square root of ρ the total energy density of space [9, Eq. 5.10 p. 173]:

$$c = \frac{k}{\sqrt{\rho}} \tag{2.1}$$

In this expression k is a constant and in the denominator ρ is a sum $\rho = \rho^* + \rho_s + \rho_E + \dots$ of $\rho^* =$ the gravitational energy density due to the faraway stars and galaxies, ρ_s = the gravitational energy density due the Sun and ρ_E = the gravitational energy density due to

Earth and any other source of energy density gravitational or otherwise.

The energy density of space ρ_B and ρ_E associated with the presence of static magnetic *B* and electric *E* fields are given by [10, p. 652 and p.772]:

$$\rho_B = \frac{1}{2\mu_o} B^2 \tag{2.2}$$

And

$$\rho_E = \frac{1}{2} \varepsilon_o E^2 \tag{2.3}$$

Where μ_0 is the magnetic permeability and \mathcal{E}_o is the electric permittivity of free space.

The gravitational energy density ρ (J/m³) of space associated with a gravitational field g (m/s²) due to a mass at a point a distance r from the centre of the mass, may be written in analogous fashion in terms of the gravitational field g (Gravitational acceleration)

 $g = GM_s / r^2$ as

$$\rho_g = \frac{1}{2} (\frac{1}{4\pi G}) g^2 \tag{2.4}$$

So that the energy density is given by: [9, p 163]

$$\rho = \frac{GM^2}{8\pi r^4} \tag{2.5}$$

With G the Universal constant of gravitation and M the gravitational mass of the body.

If the location considered is the surface of the Earth, strictly speaking, relation (2.1) should contain in the denominator the gravitational energy density due to all the other planets. However, their contribution is negligible due to the $1/r^4$ factor in the energy density. At this point it is useful to introduce the magnitudes of

the quantities of the energy density of space of several bodies. Using relations (2.2) - (2.5) the values are collected in Table 1. Please notice the wide range of the magnitudes of the energy density.

Table 1. Values of the energy density of space at the surface of Earth due to the far away stars and galaxies, mass of the Sun, Earth, the Moon and other planets and the energy density due to "strong" electric and magnetic fields achievable in a laboratory environment. All astronomical data are from NASA sources such as [11]

Source of energy density	Symbol	Energy density due to source at:	Magnitude (Joules/m ³)	Reference	Index of refraction (condition) (% Change)
Far away Stars	$ ho^*$	Earth	1.094291 x	J. Cespedes-	1.0
and Galaxies			1015	Cure p.279	(On Earth surface)
				[9],	(0,0 %)
Sun	ρ_{s}	Earth	$2.097 \text{ x } 10^4$	[12,13]	
Earth	${oldsymbol{ ho}}_{ m E}$	Earth surface	5.726 x 10 ¹⁰	[12]	

Moon	$ ho_{ m Moon}$	Earth	6.57 x 10 ⁻¹	[12]	
Jupiter	$oldsymbol{ ho}_{ ext{Jup}}$	Earth	1,91 x 10 ⁻²	†	
Venus	$ ho_{ m Ven}$	Earth	2,14 x 10 ⁻⁵	†	
Mars	$ ho_{ m Mar}$	Earth	2,91 x 10 ⁻⁸	†	
Magnetic Field	$\rho_{\rm B}$	Magnet gap	3,98 x 10 ⁵	*	1,0000000072721
2 Tesla (20 kG)	-				(in magnet gap)
					(7,2E-08 %)
Electric field	${oldsymbol{ ho}}_{ m Elect}$	Plate's gap	$1,77 \ge 10^3$	*	1,000000000002
(10 kV in 1 mm	-				(in capacitor gap)
gap)					(2,0E-13 %)

* Calculated in section 5.3. † Calculated with Eq. (2.5) at closest position to the planet

If we take relation (2.1) as describing the speed of light c in the surface of Earth and c' as the speed of light in another place where ρ' the total energy density of space is different, we can assume that the speed of light in that region is given by an expression analogous to (2.1), namely:

$$c' = \frac{k}{\sqrt{\rho'}}$$

Then we may assign an index of refraction n to quasiempty space such that n = 1 in vacuum space on the Earth surface with a null magnetic or electric fields, where c has the value 2,99792458 x 10⁸ m/s as is currently accepted, and assign an index n' (relative to the index n = 1 on the surface of Earth), to vacuum space where the energy density ρ' is different.

Defining n' as usual [10, p. 862] by n'=c/c', the use of relation (2.1) for c and for c' results in a relation where the constant k drops away and the index of refraction n' is given by

$$n' = \frac{\sqrt{\rho'}}{\sqrt{\rho}} = \frac{\sqrt{\rho'}}{\sqrt{\rho^* + \rho_S + \rho_E}}$$
(2.6)

3. Evidence for the theory.

In the following two sections are shown the evidence in support of the Cespedes-Cure hypothesis. The evidence consists of two measurements of ρ^* the energy density due to far away stars and galaxies performed by very different procedures which give the same result.

3.1 Measurement of ρ^* with the observation of eclipses of the Sun.

Jorge Cespedes-Cure in his book [9, p 297] calculates the "Cosmic energy density" ρ^* which is the gravitational energy density due to the far away stars and galaxies en the vicinity of the solar system. His value is:

$$\rho^* = 1,094291 \text{ x } 10^{15} \text{ (J/m}^3) \text{ or } (\text{N/m}^2).$$
 (3.1)

In this section we describe briefly how this quantity was obtained.

To arrive at this value he makes use of what we have called the Cepedes-Cure hypothesis above, a contention which follows from the work of Green and McCullogh as mentioned by E. Whittaker [14]. The calculation of ρ^* is based on a study of starlight deflection by the Sun which considers all observations during eclipses up to 1974. The data pertains to 297 star deflections resulting from nine groups of observations during six solar eclipses. The data collected and tabulated by Prof. P. Merat from Paris [15] results in an empirical law which relates the distance from the Sun (in units of the Sun's radius R_o), and the average measured starlight deflections in (") seconds of arc.

Cespedes-Cure studies the starlight deflection by the Sun's gravitational energy field in an alternative way to the accepted General Relativity explanation. He considers it a refraction phenomenon in which the index of refraction n' in the vicinity of the Sun creates a spherical lens with fuzzy edges producing the observed starlight deflections. An expression for the index of refraction as a function of the Sun's radius n(r) is obtained from expression (2.6) above in which the value of ρ^* is a parameter. Expressions derived from n(r) and Snell's law are used to fit the deflection to the empirical Merat's law from which the best value of ρ^* that fits the starlight deflections is calculated.

The following Table 2 shows the measured values of starlight deflections as tabulated by Merat and starlight deflections calculated by Cespedes-Cure with the best value of ρ^* on the assumption of the index of refraction change by the gravitational energy density of the Sun. The fit to the experimental data obtained by Cespedes-Cure is better than the prediction of General Relativity Theory calculated by the expression $\delta = 4GM_o / r$, where M_o is the mass of the Sun, G is Newton's Constant of Gravitation and $r = mR_o$ is expressed as a multiple m of the solar radius R_o

3.2 Measurement of ρ^* with the Pioneer anomaly

The Pioneer anomaly is an anomalous behavior in the movement of the Pioneer spacecraft and several other spacecraft, reported by NASA which exhibit acceleration towards the Earth in excess of the prediction of Newtonian gravitational theory. The Pioneer anomaly was explained based on the previous theory [12, 13]. The reported numerical values of the anomaly was used to calculate the gravitational energy density of space ρ^* due to the far away stars and galaxies. The value obtained:

$$\rho^* = 1.0838 \text{ x } 10^{15} \text{ Joule/m}^3 \tag{3.2}$$

essentially coincides with the value Eq. (3.1) obtained by Cespedes-Cure as reported above. In the following we

review very briefly what is the Pioneer anomaly (omitting technical detail) and how the value of ρ^* was deduced.

The Pioneer spacecraft were launched in 1972 (Pioneer 10) and 1973 (Pioneer 11). They had as mission the study of the Solar system, particularly Jupiter and Saturn during their fly-by and to head out into space away from the Sun after their fly-by off Saturn in a direction approximately in the plane of the ecliptic.

Table 2. Data used to calculate the value of ρ^* [9, p 275], predicted starlight deflection δ_{C-C} and comparison with astronomical measurements as reported by Merat [15].

Row	r	Merat	Merat	Cespedes-Cure	Merat
	(R _o units)	$(\delta \pm \Delta \delta)$	$(\delta - \Delta \delta)$	$\delta_{\scriptscriptstyle C-C}$	$(\delta + \Delta \delta)$
1	2,09	1,02+/-0,11	0,91	0,91	1,13
2	3,12	0,67+/-0,08	0,59	0,73	0,75
3	4,02	0,58+/-0,04	0,54	0,58	0,62
4	5,10	0,40+/-0,07	0,33	0,44	0,47
5	6,06	0,41+/-0,04	0,37	0,35	0,45
6	7,11	0,31+/-0,04	0,27	0,27	0,35
7	7,84	0,24+/-0,04	0,20	0,23	0,28
8	9,51	0,20+/-0,06	0,14	0,16	0,26
9	11,60	0,16+/-0,03	0,13	0,11	0,19

Due to the long nature of the mission they were provided with long lived instrumentation and very accurate telemetering systems. These systems proved to be extremely reliable. Pioneer 10 reached a distance from the sun of 75 AU on 11 February 2000. Contact was established on the 30th anniversary of launch: March 2nd, 2002. Quality data were received in a test on March 11th, 2002. At that time the signal took 21 hours to go and return! The speed of recession from the Sun was about 11 km/s.

With Pioneer 11, the radio continued working until the 1st of October 1990 when coherent Doppler signals were received. The spacecraft was then at 30 AU from the Sun. Ranging was accomplished by timing go and return signals, and speed by the use of the Doppler effect. Earth station sends a signal f = 2.295 GHz and Pioneer retransmits the signal back. The received signal is compared with the sent signal.

It is a double Doppler shift: $\Delta f = f_E(2\nu/c)$ (first order). An unmodelled acceleration towards Earth was detected when the spacecraft were at about 5 to 10 AU

detected when the spacecraft were at about 5 to 10 AU and thereon at greater distances [16]. The anomaly is a small excess acceleration towards the Sun: ≈ 8.65 Angstrom/s² ! The anomaly has been detected in other spacecraft flying normal to the plane of the ecliptic:

Galileo and Ulysses and in Spacecraft NEAR and ESA's Rosetta, as shown in Table 3.

Since its detection NASA made diverse attempts at explanation all of which failed: Some possible causes considered were: solar wind, radiation pressure, thermal emission (²³⁸Pu power source on board), gas leaks, electronic signal problems, software problems, modeling problems. Finally they suggested the possibility of new physics. A recent paper by Turyshev and collaborators (2012) [17] pretends to explain the Pioneer Anomaly as due to non-symmetrical heat emission. This effect had been rejected as an explanation in a detailed previous NASA report [18]. In the (2012) paper a two parameter model of the unsymmetrical heat emission is made. (Eq. (1)) The two parameters are adjusted to minimize the residuals of comparison with the measured anomalous acceleration. In this way the momentum due to asymmetrical heat emission is artificially made to be exactly the value required to account for the anomalous acceleration. Clearly any other values of the parameters, except these ad-hoc values, would fail to explain the anomalous acceleration in the Pioneer spacecraft. Additionally the unsymmetrical heat emission arguments are not valid for the anomaly that has been observed in the other spacecraft.

Papers have been published with various attempts at explanation (Many in ArKiv) including calculations within General Theory of Relativity or the suggestion of Dark matter. The latter is incompatible with the accurate Newtonian prediction of the movement of bodies in the solar system. Reported values of the anomalous acceleration suffer from very considerable scatter compatible with the difficulty of the measurement and the small magnitude.

Table	e 3.	Deep sp	bace crafts	that hav	e shown	an anoma	lous excess	acceleratio	on towards	the Sun.
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Spacecraft	Excess acceleration $(m s^{-1})$	Comment	Reference
Ulysses	$(12 + 3) \times 10^{-10}$		[18]
Galileo,	$(8 + - 3) \times 10^{-10}$	CHASM independent analysis	[18]
Pioneer 10	$(8.09 \pm 0.20) \times 10^{-10}$		[18]
Pioneer 10	(8.65 +/- 0.03) x 10 ⁻¹⁰	CHASM independent analysis with Pioneer 10 data at about 20 AU.	[18]
Pioneer 11	$(8.56 \pm 0.15) \times 10^{-10}$		[18]
Spacecraft NEAR			[19]
ESA's Rosetta			[19]

The value of ρ^* based on the anomalous acceleration was calculated using the most accurate value reported for the Pioneer 10 of a = (8.65 +/- 0.03) x 10⁻¹⁰ at a distance of 20 AU with the assumption that the index of refraction at 20 AU should be **n**' given by the Cespedes-Cure hypothesis. Calculations [12] lead to an expression for the index of refraction **n**' based on the measured excess Doppler shift E_D and the frequency f_e used by NASA to determine, via the Doppler effect, the movement of the spacecraft.

$$n' = 1 - \frac{E_D c}{2f_e G\left(\frac{M_s}{r_{Sfar}^2} + \frac{M_E}{r_{Efar}^2}\right)}$$
(3.3)

The accurate measurement by NASA of the "Excess" Doppler shift E_D , a steady frequency drift of $(5.99 \pm 0.01) \ge 10^{-9}$ Hz/s [18, p 20] of the frequency used in the transmission to the Pioneer spacecraft of $f_e = 2295$ MHz sent from Earth [18, p 15] and the use of Eq. (3.3) constitutes an accurate measurement of n' the index of refraction at 20 AU. The calculation gives $n' = 0.9999735679^{\dagger}$ so that the effective speed of light at the

site of the spacecraft at 20 AU comes to c' = 299800382 m/s which is a minute amount larger (7924 m/s) than the speed of light (accepted) c = 299792458 m/s on Earth at 1 AU. With this value of **n**' in Eq. (2.6) ρ^* is the only unknown value since the gravitational energy densities due to the Sun at 1 AU ρ_{S1AU} and due to Earth can be calculated at Earth's surface ρ_E and at 20 AU ρ_{Sfar} , ρ_{Efar} with Eq. (2.5). Solving for ρ^* we get:

$$\rho^{*} = \frac{\frac{GM_{s}^{2}}{8\pi r_{Sfar}^{4}} + \frac{GM_{E}^{2}}{8\pi r_{Efar}^{4}} - n'^{2} \left(\frac{GM_{s}^{2}}{8\pi r_{S1AU}^{4}} + \frac{GM_{E}^{2}}{8\pi r_{E}^{4}}\right)}{n'^{2} - 1}$$
(3.4)

The value of ρ^* calculated with this expression is given above in (3.2) $\rho^* = 1.0838 \cdot x \cdot 10^{15}$ Joule/m³.

This value coincides with the value, Eq. (3.1), of $\rho^* = 1.09429 \times 10^{15}$ Joule/m³ calculated by Cespedes-Curé on the basis of an entirely different phenomenon: the bending of starlight by the Sun during solar eclipses, as shown in the previous section. Numerical values used to calculate ρ^* are given in the appendix. The equality of the values calculated by entirely different procedures is

[†] To 10 digits, although rightmost digits are not significant due to imprecision of E_D

very strong, undeniable evidence for the Cespedes-Cure hypothesis

4. Consequences

4.1 On the Doppler effect

The observed Doppler shift Δf of a known light frequency f is used to determine the velocity v, of the light source in the line of sight. The Doppler shift (in first order) is given by

$$\Delta f = f \frac{v}{c} \tag{4.1}$$

From where the star velocity is obtained as:

$$v = \frac{\Delta fc}{f} \tag{4.2}$$

If at the site of the light source the gravitational energy density is ρ' different from the gravitational energy density ρ on Earth, then the value of the index of refraction there given by relation (2.6) namely

$$n' = \frac{\sqrt{\rho'}}{\sqrt{\rho}} = \frac{\sqrt{\rho'}}{\sqrt{\rho^* + \rho_s + \rho_E}}$$

Is different than on Earth,.

Defining the index of refraction n' in reference to the index of refraction n=1 in vacuum on the surface of Earth as

$$n' = \frac{c}{c'} \tag{4.3}$$

hence

$$\mathbf{c'} = \frac{c}{\mathbf{n'}} \tag{4.4}$$

It should be mentioned that c' is in fact the speed of light that is actually measured [20] if the media where light propagates has the index of refraction n'^{b}

These relations shows that in another place where ρ has a different value from Earth and the gravitational energy density ρ' is greater than on Earth, $\rho' > \rho$, we will have that n' > 1, that is to say the effective speed of light is c' < c. This is the case if we consider a location in the neighborhood of a massive body like the sun or in the center of a galaxy with a higher gravitational energy density than on Earth. On the contrary, if the gravitational energy density ρ' is smaller than on Earth, $\rho' < \rho$, we will have that n' < 1, that is to say the effective speed of light is c' > c. This is the case of a location far away from a massive body such as in interstellar space far from the Sun or far away from the center of the Milky Way or in the rim of a spiral galaxy.

Since the Doppler effect predicts the velocity of the stars with $v = \Delta f c / f$ we measure an anomalously high star velocity,

$$v' = \Delta f c' / f \tag{4.5}$$

or overestimate the velocity in locations where the gravitational energy density ρ' is smaller than on Earth, $\rho' < \rho$ such as in interstellar space far from the Sun or far away from the center of a galaxy such as in Andromeda's rim or in the rim of any spiral galaxy.

4.2 The flat rotation curve of galaxies

Astronomical observations of the velocity of stars in spiral galaxies has shown a behavior which is not in agreement with the expected radial velocity deduced by Newton's law of universal gravitation or General Relativity Theory. A body of mass m in circular motion at a distance r around another of mass M with m <<M is affected by a centripetal force given by [21, Chapter 9, p. 137]

$$\frac{GMm}{r^2} = ma = m\frac{v^2}{r} = m\omega^2 r \tag{4.6}$$

Here G is Newton's universal constant of gravitation, v is the magnitude of the tangential velocity of m around M, $v^2 / r = \omega^2 r$ is the centripetal acceleration and ω the angular velocity.

It follows that the tangential velocity and angular velocity. Are given by

$$v = \sqrt{\frac{GM}{r}}$$
 and $\omega = \sqrt{\frac{GM}{r^3}}$ (4.7)

These predictions, which are perfectly corroborated in the solar system, are not quite followed for stars rotating far away from the galactic center as calculated from astronomical measurements of the Doppler shifts. The measurement of the velocity exceeds the Newtonian prediction. The problem is akin to a mass being rotated attached by a string at a rotation velocity such that the tension exceeds the strength of the string.

Two approaches have been followed to explain the observations:

The first approach: the existence of "Dark matter" is postulated. That is, the assumption that the galaxy has cold material which is invisible by any of the observational means we have (the whole electromagnetic spectrum) with a mass M_d such that, added to the

^b The author using a one-way system of measurement of the speed of light [20] obtained in water with n' = 1.3 the reduced speed of light c' = c/1,3, a value of 2,134 x 10^8 m/s with an error of 5,3 % from expected value.

estimated visual mass M in relations (4.6), it would accounts for the observed excess velocity above what is expected according to Newton's law of universal gravitation.

The second approach is to assume a modification of Newtonian mechanics. One such approach is the "MoND" theory of Milgrom [22, 23, 24] which we will not comment.

A third alternative is as follows: The calculations of star velocity that have lead to the flat rotation curve of galaxies are incorrect. In what follows we elaborate on this third alterative. All star motions that have been measured rely on a single phenomenon: The Doppler effect of light.

The "The flat rotation curve of galaxies" has been derived using relation (4.5) on the stars at large distances from the center of Andromeda and other galaxies [25, 26] The same relation was used by Hubble to arrive at the assumption of the *expansion of the universe* and all the concepts and theories derived from this concept. These include the theory of the Big Bang, Dark Matter, Dark Energy, and the accelerated rate of expansion of very far away stars and galaxies.

Relation (4.5) assumes that c is a universal constant in vacuum and that it has the same value as measured on the surface of Earth. Also it is assumed that it has the same value in the center of Andromeda or other galaxies as well as 10^{20} m away from the center, at the stars in the spiral wings and also in the confines of the universe. In all of these places it is assumed to have the value that has been measured, in vacuum, on the surface of Earth. We question this very strong assumption in the light of the discussion on the Cespedes-Cure hypothesis presented above. There is certainly a radial variation of the gravitational energy density in Andromeda and other galaxies due to the variable distribution of the galactic mass, such that the energy density increases as we near the galactic center and decreases with increasing distance from the galactic center. Hence there should be a radial distribution of the index of refraction which is given by Eq. (2.6), namely

$$n' = \frac{\sqrt{\rho'}}{\sqrt{\rho}} = \frac{\sqrt{\rho'}}{\sqrt{\rho^* + \rho_S + \rho_E}}$$

As discussed above this relation implies that at higher gravitational energy density ρ' the index of refraction is higher and the speed of light decreases. Conversely, the gravitational energy density decreases as we get further away from the galactic center so that the index of refraction decreases and the effective speed of light increases leading to a measurement of star velocities by the use of Eq. (4.5), of an anomalously increased star velocity which does not follow the expected Newtonian prediction.

It is convenient to observe that the Milky Way according to current astronomical observations is about 30 kpc across and that the Sun lies about 8 kpc from the center on what is known as the Sagittarius arm of the Milky Way [27]. That is Earth is radially about 1/2 of the way from the center to the galactic rim. This is a location where the contribution to the gravitational energy density due to the galactic mass of the Milky Way is certainly more important than in areas at the rim of the galaxy.

In what follows on this section we derive a relation with which it is possible to calculate the radial distribution of mass in a galaxy M(r) on the basis of the measurements of the Doppler shifts Δf , f and the derived star velocities v'(r)

Using (4.4) namely, $c' = \frac{c}{n'}$, we get that the measured velocity v' is given by

$$\mathbf{v}' = \frac{\Delta f}{f} \frac{c}{n'} \tag{4.8}$$

And using (2.6), namely $n' = \sqrt{\frac{\rho'}{\rho}}$ we obtain: $v' = \frac{\Delta f}{f} \frac{c}{\sqrt{\rho'}} \sqrt{\frac{\rho}{\rho'}}$ (4.9)

At different distances from the center of the galaxy we have different tangential velocities as required by Newtonian mechanics and different gravitational energy densities. To reflect this fact we will write $v' \rightarrow v'(r)$

and
$$\rho' \rightarrow \rho'(r)$$

So that

$$\mathbf{v}'(r) = \frac{\Delta f}{f} \frac{c}{\sqrt{\rho'(r)}}$$
(4.10)

This expression again shows that for smaller gravitational energy densities the star velocity deduced by the Doppler effect gives anomalously higher velocities than the real values as predicted by the Newtonian gravitation theory.

Accepting this hypothesis we can work in the inverse fashion: assuming as correct the Newtonian gravitational theory, and deduce by the use of relation (4.10) the radial variation of the gravitational energy density $\rho'(r)$ of a galaxy, utilizing the Newtonian expected values v(r) of the star velocities. From (4.10) we get

$$\rho'(r) = \left(\frac{\Delta f}{f}\right)^2 \left(\frac{c}{v(r)}\right)^2 \rho \qquad (4.11)$$

Since $\rho'(r)$ depends on the radial mass distribution of the galaxy M(r) we can rewrite equation (2.5) as follows:

$$\rho'(r) = \frac{G\left[M\left(r\right)\right]^2}{8\pi r^4}$$

Equating with relation (4.11) it is possible to solve for the galactic radial mass distribution M(r):

$$M(r) = \left(\frac{\Delta f}{f}\right) \sqrt{\frac{8\pi\rho}{G}} \frac{c}{v(r)} r^2 \qquad (4.12)$$

In this expression ρ is the gravitational energy density on the surface of Earth, c the speed of light c = 2,99792458 x 10⁸ m/s, $\left(\frac{\Delta f}{f}\right)$ are astronomical measurements of the spectra of a particular stars which are located at a distances r from the galactic center. This expression for the radial distribution of mass M(r) in a galaxy may be used with astronomical measurements of the spectroscopic shifts Δf , f, and the de star velocities v(r) expected from Newtonian mechanics to obtain, in an independent way, the galaxy's radial mass distribution M(r).

4.3 On Hubble's empirical law.

In this section we want to put forth some ideas regarding the astronomically observed redshifts of stars and galaxies that has lead to Hubble's law relating apparent distance from Earth and recession velocities. We will only advance plausible arguments based on the Cespedes-Cure hypothesis presented above without going into numerical calculations which would require a detailed model of the universe. Observed redshifts are assumed, via the first order Doppler effect of light, to be due to a velocity in the line of sight of the source of the light. The increased recessional velocity associated with further distance which was observed by Hubble in the 100-in telescope at Mount Wilson, has lead to the concept of an expanding universe and to the hypothesis of the cause of the expansion as due to an initial explosion called the "Big Bang". More recent collaborative observations [28, 29] with larger Earth telescopes and telescopes in Earth orbit have shown extreme values for the redshifts of very far away galaxies which do not follow the linear relation proposed by Hubble, but rather an accelerated rate of expansion. The large values of the recession velocities at the greatest distances lead Hubble himself to question the interpretation of the redshifts as true measure of recession velocities [30]. Many other authors have also guestioned this interpretation giving rise to alternative ideas on the cause of the observed redshifts. One such theory is the so called "Tired light theory": The assumption that there is loss of the energy of the light or absorption of the energy of light as it transverses the huge distances involved. [See ref. 21 Section 18.6.1, P. 355 and references therein]. The cosmological red shift can also be interpreted in an elegant and rigorous way [9, see Sec. 6.8] but we will not comment on this interpretation here.

As discussed above all radial, or line of sight, star motions that have been reported rely on a single phenomenon: the Doppler effect of light. The observed Doppler shift Δf of a known light frequency f is used to determine the velocity, in the line of sight, v of the light source by Eq. (4.5) above, namely: $v' = \Delta f c' / f$. We have shown above that the index of refraction n' in a region of space with a gravitational energy density value ρ' different from the value on Earth surface ρ is given by Eq. (2.6) namely

$$n' = \frac{\sqrt{\rho'}}{\sqrt{\rho}} = \frac{\sqrt{\rho'}}{\sqrt{\rho^* + \rho_s + \rho_E}}$$

The gravitational energy density is given by Eq. (2.5) above namely:

$$\rho = \frac{GM^2}{8\pi r^4}$$

This suggests that a possible interpretation of the observed extreme cosmological redshifts may be related to a variation of the gravitational energy density with distance. In particular if we assume as a model a universe with expansion but with its mass distribution limited in extension, it is reasonable to assume that the volumetric mass density of stars and galaxies decreases radially towards the limit, and that the gravitational energy density decreases as we go further and further away towards the limiting regions. Eq. (2.6) then predicts that the index of refraction n' becomes smaller and smaller as we go further and further away. Hence the effective speed of light c' increases due to Eq. (4.4) namely c' = c/n'leading to higher, anomalous and overestimated values of the derived star velocities. We do not have an independent way to determine star velocities of far away stars and galaxies as in the case of the stars in the rim of rotating galaxies. Hence we should rely on experimental measurements that confirm or falsify the Cespedes-Cure hypothesis to consider this plausible explanation of the accelerated recession velocities of far away stars and galaxies.

5. Experimental proposals^c

In this section are presented three experimental proposals that would verify A. Einstein's contention discussed

[°] Not included is a pending explanation of the "Fly-by anomaly" also detected by NASA.

above and also the Cespedes-Cure hypothesis. They are presented in addition to the evidence provided by the coinciding values of ρ^* the gravitational energy density of far away stars and galaxies calculated by the interpretation of two totally different phenomena: The bending of starlight by the Sun during eclipses (Sec. 3.1 above) and by the measurements by NASA of the Pioneer anomaly (Sec. 3.2 above). They are difficult experiments but by no means impossible as shown by the numerical calculations presented.

5.1 Index of refraction between Jupiter and Earth.

The Index of refraction of the space between Jupiter and Earth can be measured by doing a modern, very accurate, measurement of the speed of light by the method used by Roemer [31]. In work during a 7 year period ending in 1676, Roemer did painstaking measurements of the times of eclipses of the moons of Jupiter. He discovered and documented seasonal changes of the times of occultation (or reappearance) of Io, the first satellite of Jupiter.[See a detailed discussion by J. H. Shea in Ref. 32]. With a correct interpretation of the reason for the observed variations in the period of rotation and times of occultation. C. Huvgens, using Roemer's data, was able to calculate for the first time the one-way speed of light from Jupiter to Earth. Using the theory explained above, the index of refraction of the space between Jupiter and Earth is expected to be slightly smaller than one. This follows from an examination of relation (2.6) namely

$$n' = \frac{\sqrt{\rho'}}{\sqrt{\rho}}$$

The gravitational energy density on the surface of Earth ρ is expected to be slightly greater than in the space between Jupiter and Earth ρ' so that the index of refraction n' is expected to be n' < 1 and the speed of light in the space between Jupiter and Earth slightly higher than c.

Making use of relation (2.5) namely

$$\rho = \frac{GM^2}{8\pi r^4}$$

We can calculate the total gravitational energy density in the surface of Earth and in the space between Earth and Jupiter by adding the contributions due to the Sun, Earth, Jupiter and the far away stars and galaxies, and then calculate values for n'. The results are shown in Table 4. The numerical results in this table reflect the $1/r^4$ nature of Eq. (2.5). The gravitational energy density decreases very fast with distance and the contributions due to the Sun, Earth and Jupiter become negligible compared to the value of ρ^* due to the far away stars and galaxies. Hence, unless we consider a region relatively near the planets such as on the planet surface or its atmosphere, in the region between Earth and Jupiter it is ho^* that

determines n' the index of refraction.

To verify the predictions of Table 4 we need to make very accurate measurements. The fractional change in the index of refraction in the space between Jupiter and Earth compared to the value on Earth surface is $2,62 \times 10^{-5}$ or just 0,00262 %. To accomplish this measurement we propose a very simple method: a variation of the use of Roemer's method using timed observations of the rotation of Jupiter's satellite Io^d. We propose to use modern instrumentation to determine the one-way speed of light. Instead of attempting measurement of the times of eclipses of the moons of Jupiter which is difficult and has very large errors and which has been used traditionally to determine the ephemeredes of Jupiter's satellites [33], a digital photographic method is proposed. The method is accessible to amateur astronomers with modest telescopes equipped with digital cameras of sufficient resolution and capable of recoding accurate time. The method is made clear with reference to Fig. 1.

Photos are taken of Jupiter and its satellite Io with widely available high resolution digital cameras. Several sets of photographs are taken at arbitrary, but carefully recorded, Universal Times (UT) (say every 15 minutes) during an observation period of a couple of days each set. The timing of the sets of data taken should span at least 6 months in order to cover different relative Jupiter-Earth distances.

For each digital photograph two values are recorded: 1) The time UT of the photograph, and 2) an accurate value of the distance of Io relative to the center of Jupiter (in pixels). The unit derived for this distance in each photo is in visual Jupiter diameters of the particular photo. The later is obtained by analysis of the digital images. With these values a plot is made of the relative position of Io (Y) as a function of time UT. This graph will look as shown in Fig. 1 and consist of experimental points which will be fitted by a sinusoidal curve. If we assume a circular orbit for Io, the fitted curve would be:

$$Y = Y_{Max} Sin(\omega t + \phi)$$
 (5.1)



^d Rangel A. and Greaves E. D. Medida del período de las lunas de Júpiter para la determinación de la velocidad de la luz en el espacio entre la Tierra y Júpiter. Manuscript for XXV Encuentro Nacional de Astronomía, Maracaibo, Venezuela (March, 2015)

Fig. 1 On the left is a simulated digital photo of Jupiter [34] including its Galilean satellites taken at time t. In the center is a simulated plot of the relative position Y of Io (in the photos) as a function of Universal Time (UT) The sinusoidal curve is a (simulated) non-linear fit to the points for a set of photos taken over the time of two Io revolutions, equivalent to about 3,5 days

With fitting parameters: the amplitude Y_{Max} , the angular

rotation velocity ω , and the phase angle ϕ of the apparent Io rotation. These parameters will allow the calculation of the speed of light. Both, ω the apparent angular rotation velocity and ϕ , the phases, are expected to vary with significant changes of the universal time or Earth-Jupiter distance. The angular rotation velocity of Io ω_o is constant. The orbital period of Io is reported as 1,769 days or 152 854 seconds [35]. However, the observed or apparent angular rotation velocity ω

changes due to the Doppler effect: with higher values when the earth moves in its orbit *towards* Jupiter and with lower values when the earth moves in its orbit *away* from Jupiter. The phase angle ϕ changes as a function of the Jupiter-Earth distance because it is affected by the variable time $t = t_o \pm \phi / \omega$ it takes for light to transverse the distance to earth at the finite speed of light. The phase angle decreases when the Jupiter-Earth distance decreases and it increases when the Jupiter-Earth distance increases with t_o the time for light to reach Earth at the nearest approach. The Jupiter-Earth distance x is obtained from

astronomical ephemerides and the Universal Time t as recorded for each photograph. Finally the slope $\partial x / \partial t$ of a plot of calculated changes in Jupiter-Earth distances x as a function of changes in time due to changes in phase angle ϕ / ω will yield the average speed of light in the Jupiter-Earth space. In order to verify the Cespedes-Cure Hypothesis it is necessary to

Table 4. Gravitational energy density, index of refraction and speed of light at different positions between Earth and Jupiter assuming, for simplicity, alignment between Jupiter, Earth and the Sun.

Distance	Gravitational energy density contribution (J/m ³)*					Index of	Speed of
to	Stars and	Sun	Earth	Jupiter	Total	refraction	light c' (m/s)
Jupiter	Galaxies					n'	
@ Earth	1,09429E+15	2,0970E+4	5,73E+10	6,112E-05	1,094E+15	1,000000000	299792458
surface							
1⁄4	1,09429E+15	1,18E+03	1,551E-07	1,9312E-4	1,094E+15	0,999973821	299800306,4
1/2	1,09429E+15	2,26E+02	9,693E-09	9,7798E-4	1,094E+15	0,999973821	299800306,4
3⁄4	1,09429E+15	7,05E+01	1,914E-09	1,5647E-2	1,094E+15	0,999973821	299800306,4
@ Jupiter surface	1,09429E+15	2,86E+01	6,058E-10	7,15E+07	1,0946E+15	1,00014112	299750157,3

* Values are given without the necessary significant figures due to printing space limitation.

achieve sufficient accuracy in the fitting parameters Y_{Max} ,

 ω and ϕ , particularly the two last ones. As shown in Table 4 the calculated index of refraction in the space between Jupiter and Earth, 0,999973821, differs little from 1, hence the predicted speed of light c' differs little from c. The change is just 0,00262 % which requires that the measurements of the rotational period of Io and the phase angles have uncertainties of less than +/- 4 s. We propose to carry out this measurement by securing the collaboration of amateur astronomers worldwide by making an appeal to participate in the measurement. Thereby a large amount of data will be gathered contributing to the accuracy of the measurements.

5.2 Index of refraction on the International Space Station (ISS)

In this section we calculate the index of refraction at the height of the ISS where an accurate instrument could be taken to measure the speed of light.

The index of refraction on the International Space Station

(ISS) n'_{ISS} may be calculated with Eq. (2.6) namely:

$$n'_{ISS} = \frac{\sqrt{\rho'_{ISS}}}{\sqrt{\rho}} = \frac{\sqrt{\rho'_{ISS}}}{\sqrt{\rho^* + \rho_S + \rho_E}}$$

Where the total gravitational energy density $\dot{\rho_{\rm ISS}}$ is given

by $\sqrt{\rho_{ISS}} = \sqrt{\rho^* + \rho_S + \rho_{E-ISS}}$, with ρ_{E-ISS} the gravitational energy density due to Earth at the height of the ISS. Since the gravitational energy density due to the far away stars ρ^* and due to the Sun ρ_S are expected to be the same as on the surface of Earth relation (2.6) yields

$$n'_{ISS} = \frac{\sqrt{\rho'_{ISS}}}{\sqrt{\rho}} = \sqrt{\frac{\rho^* + \rho_S + \rho_{ISS}}{\rho^* + \rho_S + \rho_E}}$$
 (5.2)

Making use of relation (2.5) we get for ρ_{E-ISS}

$$\rho_{E-ISS} = \frac{GM_E^2}{8\pi r_{ISS}^4} = \frac{GM_E^2}{8\pi (r_E + h_{ISS})^4}$$
(5.3)

Where M_E is the mass of Earth, r_E is the radius of Earth and h_{ISS} is the height of the ISS (333,25 Km). Substitution of (5.3) into (5.2) gives the index of refraction at the ISS as

$$n'_{ISS} = \sqrt{\frac{\rho^* + \rho_S + \frac{GM_E^2}{8\pi (r_E + h_{ISS})^4}}{\rho^* + \rho_S + \rho_E}}$$
(5.4)

From where the speed of light at the ISS may be obtained with Eq. (4.4)

$$\mathbf{c}_{ISS}' = \frac{C}{\mathbf{n}_{ISS}'} \tag{5.5}$$

. In order to verify the Cespedes-Cure hypothesis by measuring the speed of light in the ISS, very accurate measurements need to be made. The value of the index of refraction at the ISS predicted by Eq. (5.4) is 0,99997869369. And the corresponding speed of light in the ISS predicted by Eq. (5.5) is 299798845,6 a difference from accepted c of only 6387,6 m/s or 0,00213 % change. These values indicate the accuracy requirement on an instrument designed to be taken to the ISS to verify or falsify the A. Einstein contention and the Cespedes-Cure hypothesis.

5.3 Index of refraction due to an electric and magnetic field.

The index of refraction n_B ' on the surface of Earth in the space where there is a magnetic field B may be calculated with the use of Eq. (2.6).

$$n_{B}' = \frac{\sqrt{\rho_{B}'}}{\sqrt{\rho}} = \frac{\sqrt{\rho_{B}'}}{\sqrt{\rho^* + \rho_{S} + \rho_{E}}}$$
(5.6)

Where ρ_B ' is the total energy density due to gravitation with the addition of the energy density due to the magnetic field. The contributions of a magnetic or electric fields are given by Eqs. (2.2) and (2.3) namely

$$\rho_B = \frac{1}{2\mu_o} B^2 \text{ and } \rho_e = \frac{1}{2} \mathcal{E}_o E^2$$

Hence, the total energy density ρ_B' at the site of the magnetic field is the sum of the gravitational energy densities at the surface of Earth due to the far away stars

and galaxies, the contributions of the Sun and Earth plus the contribution due to the magnetic field:

$$\rho_{B}' = \sqrt{\rho^{*} + \rho_{S} + \rho_{E} + \rho_{B}} = \sqrt{\rho^{*} + \rho_{S} + \rho_{E} + \frac{B^{2}}{2} \mu_{0}}$$

Substitution into (5.6) gives

$$n_{B}' = \frac{\sqrt{\rho^{*} + \rho_{S} + \rho_{E} + B^{2}/2\mu_{0}}}{\sqrt{\rho^{*} + \rho_{S} + \rho_{E}}} = \sqrt{1 + \frac{B^{2}/2\mu_{0}}{\rho^{*} + \rho_{S} + \rho_{E}}}$$
(5.7)

In analogous fashion, the index of refraction on the surface of Earth in the space where there is an electric field E is given by:

$$n_E' = \frac{\sqrt{\rho^* + \rho_S + \rho_E + \frac{1}{2}\varepsilon_o E^2}}{\sqrt{\rho^* + \rho_S + \rho_E}} = \sqrt{1 + \frac{\frac{1}{2}\varepsilon_o E^2}{\rho^* + \rho_S + \rho_E}}$$
(5.8)

Numerical values calculated with Eqs. (5.7) and (5.8) have been included in Table 1. Examination of these values shows that measurements are more convenient with the use of a magnetic field than with an electric field by over 3 orders of magnitude. The small values of the change of the index of refraction due to the presence of a magnetic field shows the tall order that A. Einstein was asking Peter Kapitza in 1930 [3].

In order to measure the change of index of refraction produced by a magnetic field we propose to use a variant of the procedures reported to detect the lower limit of the charge of the photon. [5, 6]. A short outline of the proposal follows^e. See Fig. 2. A beam of light from a He-Ne laser is made to transverse the gap of a strong magnet (~2 Tesla) in an off-center fashion. The magnet gap is akin to a disc lens of index of refraction n_B ' causing a deflection of the

beam. To detect the deflection the beam is made to go through a special mirror system that through several internal reflections magnifies by several orders of magnitude the beam deflection. The beam impinges on a quadrant photo detector to determine its possible position change. In order to filter expected noise, a phase sensitive detection system is used. The magnet supply is modulated and the modulation frequency, as well as photo detector output, is used as inputs to a lock-in amplifier.

^e Greaves, E. D., Mikoss I., Rodríguez A. M., Aloma E. and Sajo-Bohus L. The speed of light in a magnetic field. Manuscript in preparation (2015)



Fig. 2 Schematic diagram of the instrument designed to detect the possible change of the index of refraction of space produced by the presence of a magnetic field. FG Function Generator; MPS Magnet Power Supply

The expected beam deflection may be calculated considering the field of the magnet as a spherical lens with refraction index n'. As a first approximation, the fuzzy nature of the lens borders is ignored. The angular deflection produced by this lens can be calculated with the equations of classical optics [36].

(

$$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2} \right]$$
5.9)

Where *f* is the focal length of the lens, *n* is the refractive index of the lens material, R_1 is the radius of curvature of the lens surface closest to the light source, R_2 is the radius of curvature of the lens surface farthest from the light source, and *d* is the thickness of the lens (the distance along the lens axis between the two surfaces).

For a light beam that transverses the lens by the center of the magnet gap there is no deflection. The maximum deflection is obtained by a beam near the rim of the gap (like a beam near the rim of a lens). In our case the lens is spherical that is, both curvature radii are equal $R_1 = R_2 = R$. The lens has a thickness d = 2 R so that equation (5.9) becomes:

$$\frac{1}{f} = (n'-1) \left[\frac{2}{R} - \frac{(n_B'-1) 2R}{n_B' R^2} \right]$$

This simplifies to:

$$\frac{1}{f} = (n_B' - 1) \frac{2}{n_B' R}$$
(5.10)

The deflection angle of a ray that passes by the rim of the magnet gap may be calculated in rad by dividing the radius of the gap R by the focal distance f given by (5.10):

$$\theta = \frac{R}{f}.$$
(5.11)

Substituting (5.10) we get:

$$\theta = \frac{R}{f} = R(n_B'-1)\frac{2}{n_B'R}$$
$$\theta = 2\left(1 - \frac{1}{n_B'}\right)$$
(5.12)

In these relations n_B' is obtained with (5.7) when in the gap there is a magnetic field **B**

To estimate the value of n_B' we used values of the gravitational energy density taken from Table 1: $\rho^* = 1.09429 \times 10^{15}$ Joules/m³, energy density due to the far away stars and galaxies; $\rho_S = 2.097 \times 10^4$ Joules/m³, energy density due to the Sun on Earth's surface and $\rho_E = 5.726 \ 10^{10}$ Joules/m³, energy density due to Earth. The magnitudes show that the contributions to n' due to Earth and the Sun are negligible.

As shown in Table 1 with a magnetic field of 2 T the index of refraction in the magnet gap is 1,00000000072721 which is a change of 7,27 x 10^{-8} % compared with the index with no field. Substitution in Eq. (5.12) we get for the deflection angle $\theta = 1,45 \times 10^{-9}$ radians, a very small deflection indeed. The possibility of measuring a deflection angle of this small magnitude is shown by the system reported by A. Hankins, C. Rackson, and W. J. Kim [6]. They claim that an angular sensitivity of the order of 1 x 10^{-10} rad is possible by using a mirror levering system of 100 m and a phase sensitive detection of the beam position at a quadrant photo detector with a deflection sensitivity of +/- 13 nm.

In Fig. 2 the mirror system is a design to increase the deflection angle. It consists of tandem curved cylindrical mirrors where light impinges after multiple reflections on the quadrant photodetector.

Conclusion

In this work we elaborate on Einstein's 1930 contention that the magnetic energy density affects the propagation of light, expanding it to include the effect on the propagation of light by the energy density of space due to gravitational masses such as the Sun, Earth or far away stars and galaxies.

We show that two widely different experimental measurements: the bending of light by the Sun during eclipses and the apparently anomalous movement of the Pioneer spacecraft, both lead to the same value of the gravitational energy density of space due to the far away stars and galaxies. These results, which cannot be due to a coincidence, give strong support to the A. Einstein contention of 1930 that a magnetic field affects light propagation and lends credibility to the Cespedes-Cure hypothesis that the sums of gravitational, electric and magnetic energy densities determine the index of

refraction of quasi-empty space. This hypothesis has far reaching consequences for physics in general and for cosmology in particular. We show how the "Flat rotation curve of galaxies" might be explained as due to an incomplete interpretation of the Doppler shifts of stars in the rim of galaxies produced by a radial variation of the index of refraction of space. We also show how this hypothesis might explain deviations of Hubble's linear law exhibited by galaxies at the furthest distances with the additional assumptions of a limited universe whose matter density decreases with distance.

Finally we provide calculations for three feasible experiments that may provide additional evidence to the A. Einstein's contention and the Cespedes-Cure hypothesis. They all require very accurate measurements of the index of refraction of space: At the International Space Station, in the space between Jupiter and Earth and in the field of a strong magnet. This last one is an experiment that was proposed by A. Einstein 85 year ago.

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Appendix

Numerical values used in calculations

It is convenient for readers to have the numerical values of quantities that were used in the calculations. They are collected in the following table. Calculated results are affected by the limited number of significant figure of the variables used; however I have chosen to express numerical results, in this work, with more than the recommended number of significant figures.

Variable	Value
G gravitation constant	$6.67300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Sun's mass	$1.98892 imes 10^{30} \mathrm{Kg}$
Earth's mass	5.976 x 10 ²⁴ Kg
Frequency transmitted from Earth to the Pioneer spacecraft	2.295 x 10 ⁹ Hz
Frequency shift (at 20 AU)	5.99 x 10 ⁻⁹ Hz/s
Distance from Earth (19 AU)	2.84236 x 10 ¹² m
Distance from Sun (20AU)	2.99196 x 10 ¹² m
Radius of the Earth	6.378140 x 10 ⁶ m
1 AU	$1.49598 \ 10^{11} \ \mathrm{m}$
$ ho^*$ Value obtained by	$1.09429 \text{ x } 10^{15} \text{ Joule/m3}$
Jorge Cespedes-Cure [9]	
The index of refraction calculated using the Pioneer anomaly at $R = 20$ AU:	n'= 0.999973567
Speed of light On Earth at 1 AU (accepted):	c = 299792458 m/s
<i>c</i> ' Becomes at 20 AU:	c' = 299800382 m/s
ρ^* Value obtained by	1.0838×10^{15} Joule/m ³
E. D. Greaves [12]	

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³ Kapitza Peter relates the event as: "In the 30s, in the Cavendish laboratory I developed a method for obtaining superior magnetic field intensities by one order of magnitude to those known. In a conversation Einstein tried to convince me that I study experimentally the influence of a magnetic field on the speed of light. These experiments had been carried out without any effect having been achieved. With my magnetic fields, one could increase the accuracy of the measurement by two orders of magnitude since the effect should depend on the square of the magnetic field intensity. I objected to Einstein that according to existing representations about electromagnetic phenomena, it was not evident from where one could expect such measurable effect. When not finding the possibility of demonstrating the fundament of such experiments, Einstein at last told me: "I think that the dear God ("der liebe Gott") could not create the world in such a manner that the magnetic field does not influence the speed of light". Of course this is an argument against which it is difficult to discuss"

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